



Monday cup #5 – Solution

Posted on: April 29, 2019

Due on: may 7, 2019



Problem

In preparing to write an examination, **Erin MacDuff** made the following observations: the exam had twenty questions, he estimated that he would spend $1/10$ hours per question, and the exam would take him 120 minutes to complete. However, during the actual examination, **Erin** discovered some difficult questions which each required $1/4$ hours to complete. He also discovered some questions which were much easier than he expected and only took him $1/30$ hours per question to complete. Seven of the questions, however, still required $1/10$ hours. each to complete. Surprisingly, **Erin** was still able to complete the exam in 120 minutes. How many of the twenty examination questions did **Erin** find difficult?

Solution

Solution 1: Systematic Trial

Since 7 of the questions required 6 minutes each to complete, it took **Erin MacDuff** $7 \times 6 = 42$ minutes to

complete these questions. The total exam took 2 hours or 120 minutes. He had $120 - 42 = 78$ minutes to complete $20 - 7 = 13$ questions.

Let d represent the number of difficult questions and e represent the number of easier questions. We know that $d + e = 13$.

Since each difficult question took 15 minutes, it took $15d$ minutes to complete all of the difficult questions. Since each easier question took 2 minutes, it took $2e$ minutes to complete all of the easier questions. Since **Erin's** total remaining time was 78 minutes, $15d + 2e = 78$ minutes.

At this point we can pick values for d and e that add to 13 and substitute into the equation $15d + 2e = 78$ to find the combination that works. (We can observe that $d < 6$ since $15 \times 6 = 90 > 78$. If this were the case, then e would have to be a negative number.)

If $d = 3$ then $e = 13 - 3 = 10$. The time to complete these would be $15 \times 3 + 2 \times 10 = 45 + 20 = 65$ minutes and he would complete the exam in less than 2 hours.

If $d = 4$ then $e = 13 - 4 = 9$. The time to complete these would be $15 \times 4 + 2 \times 9 = 60 + 18 = 78$ minutes and he would complete the exam in exactly 2 hours.

Therefore, **Erin** found 4 of the questions to be more difficult and time-consuming than he expected.

Solution 2 involves algebra and equation solving.

Solution 2: Using Algebra and Equations

Since 7 of the questions required 6 minutes each to complete, it took **Erin** $7 \times 6 = 42$ minutes to complete these questions. The total exam took 2 hours or 120 minutes. He had $120 - 42 = 78$ minutes to complete $20 - 7 = 13$ questions.

Let d represent the number of difficult questions and $(13 - d)$ represent the number of easier questions.

Since each difficult question took 15 minutes, it took $15d$ minutes to complete all of the difficult questions. Since each easier question took 2 minutes, it took $2(13 - d)$ minutes to complete all of the easier questions.

Since **Erin**'s total remaining time was 78 minutes,

$$15d + 2(13 - d) = 78$$

$$15d + 26 - 2d = 78$$

$$13d + 26 = 78$$

Subtracting 26 from both sides: $13d = 52$

Dividing both sides by 13: $d = 4$

Therefore, **Erin** found 4 of the questions to be more difficult and time-consuming than he expected.

There were correct solutions from Anamaria Megrelidze (Georgia, the country), Tornike Tsulukidze (Georgia, the country), Dato Tavdgiridze (Georgia, the country), Nika Darsalia (Georgia, the country) and Gela Tsetskhladze (Georgia, the country).

The prize was split between Tavdgiridze and Darsalia

Rules

1. Anyone is eligible to participate. Each solution is to be the work of one individual without any input from faculty or others. An answer must be accompanied by appropriate justifications to be considered correct.
2. The solution is to be submitted with the solver's name, email, year in school (if applicable), local phone number, and local address. If you are submitting this for possible credit in a class, include your class number and instructor's name.
3. The solution is to be typed or legibly written. Solutions must be submitted to the by 2 p.m. on the due date.
4. Entries will be graded on clarity of exposition and elegance of solution. An award of **GEL10** will be given for the best correct solution. In the case of a two-way tie, the award will be split. If there are more than two best solutions, a drawing will be held to determine two award winners.
5. Graduate students, faculty, and members of the general public are encouraged to submit solutions, but they will not be considered.

გზიპაბათობ თაბო, кубок понедельника, Monday cup, Coppa del lunedì, Coupe du lundi
Solution for this problem can be submitted proveweek@gmail.com