



## Monday cup #23- Solution

Posted on: September 2, 2019

Due on: September 15, 2019



### Problem

**Question:** Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the eighth term of each sequence is  $N$ . What is the smallest possible value of  $N$ ?

(Difficulty: 4)

### Solution to Problem:

Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the eighth term of each sequence is  $N$ . What is the smallest possible value of  $N$ ? (Difficulty: 4) Let the first two terms of the first sequence be  $X_1$  and  $X_2$  and the first two of the second sequence be  $Y_1$  and  $Y_2$ . Computing the eighth term in terms of the variables, we get  $N = 8X_1 + 13X_2 = 8Y_1 + 13Y_2$ . Taking both sides modulo 13, we get  $8X_1 \equiv 8Y_1 \pmod{13}$  or  $X_1 \equiv Y_1 \pmod{13}$ . To minimize, we let the residue be 0 and  $X_1 = 0$ . Then,  $Y_1 = 13$  (since that's the next smallest multiple of 13). Because the sequences are non-decreasing,  $Y_2 \geq Y_1$ . To minimize, we let  $Y_2 = Y_1 = 13$ . Therefore, the smallest possible value of  $N$  is  $8 \cdot 13 + 13 \cdot 13 = 273$ .

## There was no correct solution to problem 23

### Rules

1. Anyone is eligible to participate. Each solution is to be the work of one individual without any input from faculty or others. An answer must be accompanied by appropriate justifications to be considered correct.
2. The solution is to be submitted with the solver's name, email, year in school (if applicable), local phone number, and local address. If you are submitting this for possible credit in a class, include your class number and instructor's name.
3. The solution is to be typed or legibly written. Solutions must be submitted to the by 2 p.m. on the due date.
4. Entries will be graded on clarity of exposition and elegance of solution. An award of **GEL10** will be given for the best correct solution. In the case of a two-way tie, the award will be split. If there are more than two best solutions, a drawing will be held to determine two award winners.
5. Graduate students, faculty, and members of the general public are encouraged to submit solutions, but they will not be considered.