



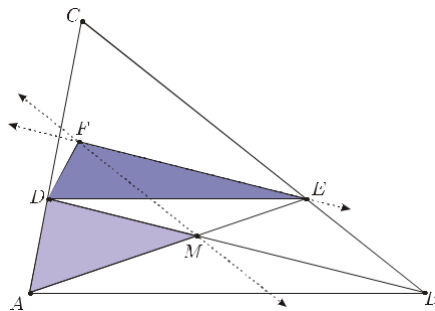
Monday cup #17- Solution

Posted on: July, 22,2019

Due on: July, 28, 2019

Problem

Question: Let ABC be an arbitrary triangle. Let D and E be points on \overline{AC} and BC respectively such that \overline{DE} is parallel to \overline{AB} . Let M be the intersection of \overline{BD} and \overline{AE} . Let F be the intersection of the line through M parallel to \overline{BC} , and the line through E parallel to \overline{BD} . Prove that the triangles DEF and AMD have the same area.



Solution by Takumi Saegusa:

We have a theorem that says that triangles on equal bases and between the same parallels are equal in area. (a)

Since the triangles DEF and EFM have the line segment EF in common and since the line segments DM and FE are parallel, the areas of the triangles DEF and EFM are the same by (a). (1)

Since the triangles EFM and BFM have the line segment FM in common and since the line segments BE and MF are parallel, the areas of the triangles EFM and BFM are the same by (a). (2)

Since the triangles BEM and BFM have the line segment BM in common and since the line segment BM and EF are parallel, the areas of the triangles BEM and BFM are the same. (3)

Since the triangles ABE and ABD have the line segment AB in common and since the line segments AB and DE are parallel, the areas of the triangles ABE and ABD are the same. (4)

Then

$$\begin{aligned}
 & \text{(the area of the triangle } AMD) \\
 = & \text{(the area of the triangle } ABD) - \text{(the area of the triangle } ABM) \\
 = & \text{(the area of the triangle } ABE) - \text{(the area of the triangle } ABM) \text{ by (4)} \\
 = & \text{(the area of the triangle } BEM) \\
 = & \text{(the area of the triangle } BFM) \text{ by (3)}
 \end{aligned}$$

- = (the area of the triangle EFM) by (2)
- = (the area of the triangle DEF) by (1). Q.E.D.

There was no correct solution to problem 17

Rules

1. Anyone is eligible to participate. Each solution is to be the work of one individual without any input from faculty or others. An answer must be accompanied by appropriate justifications to be considered correct.
2. The solution is to be submitted with the solver's name, email, year in school (if applicable), local phone number, and local address. If you are submitting this for possible credit in a class, include your class number and instructors name.
3. The solution is to be typed or legibly written. Solutions must be submitted to the by 2 p.m. on the due date.
4. Entries will be graded on clarity of exposition and elegance of solution. An award of **GEL10** will be given for the best correct solution. In the case of a two-way tie, the award will be split. If there are more than two best solutions, a drawing will be held to determine two award winners.
5. Graduate students, faculty, and members of the general public are encouraged to submit solutions, but they will not be considered.

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Solution for this problem can be submitted proveweek@gmail.com