Monday cup #16- Solution

Posted on: July, 15,2019
Due on: July, 21, 2019

Problem

Question: You have 12 "moneta" that look and feel identical. However, one of the "moneta" is either slightly lighter or slightly heavier than the rest. You cannot tell which"moneta" is the odd one out by handling them, and you do not know if that "moneta" is lighter or heavier.

You have a set of old-fashioned balance scales. The scale with more weight in it will tip down, and the scale with less weight in it will tip up.

How do you figure out which "moneta" is the odd one out, and whether it is lighter or heavier, in just three weighings?

Every time you weigh a set of cue balls, you gain information. The balls can be separated into four categories:

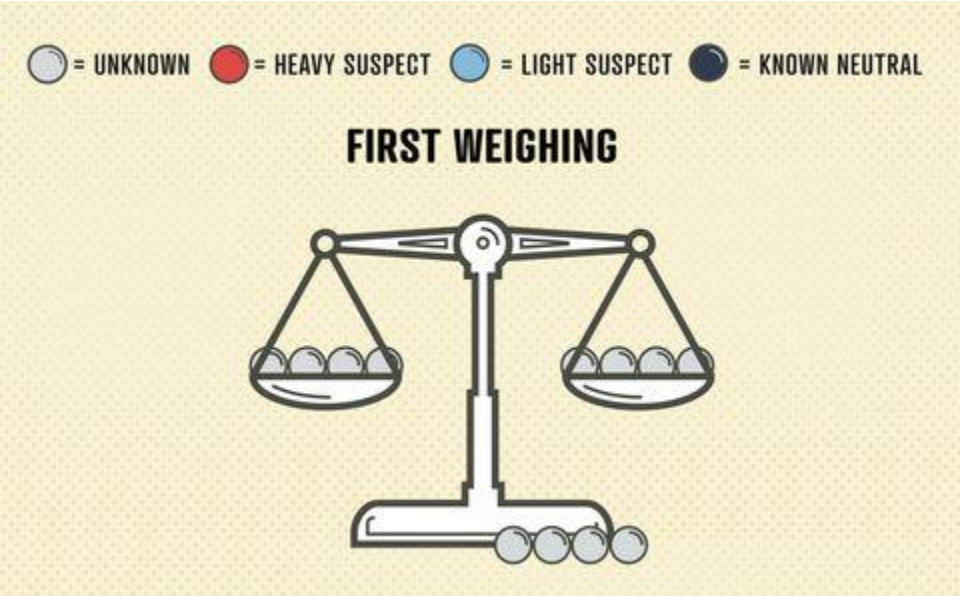
1. **"moneta" that you know nothing about, which I call Unknowns.**
2. "moneta" **that you know are *not* the odd ball out, which I call Known Neutrals.**

3. "moneta" **that could be lighter than the rest, or Light Suspects.**
4. "moneta" **that could be heavier than the rest, or Heavy Suspects.**

With this in mind, check out the original question again here, and each time you weigh a set of cue "moneta" , keep track of which category each "moneta" falls into. Then think of clever ways to weigh combinations of the balls that will give you the most information possible. On to the solution.

Solution

First, you weigh four cue balls against four cue "moneta" . One of three things can happen. The scale can stay balanced, it can tip to the left, or it can tip to the right.



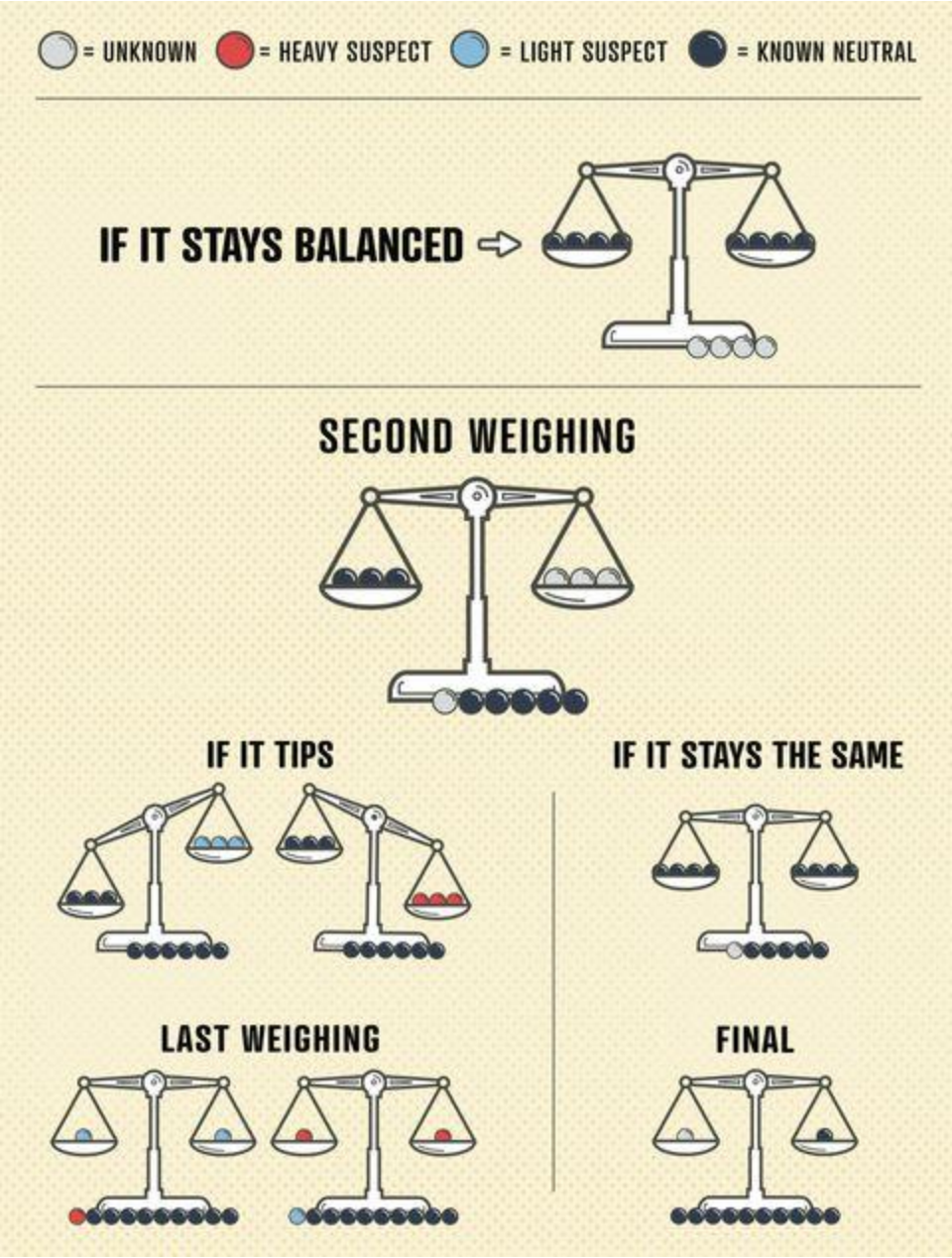
Let's consider the first possibility, that the scale stays balanced on the first weighing of four vs. four. You now know that all eight "moneta" on the scale are Known Neutrals. The four remaining off the scale are still Unknowns, and one of those four "moneta" is either slightly heavier or slightly lighter than the rest. We need to figure out which one, and if it's light or heavy. There are a few different ways you can set up the second weighing to arrive at the solution, but the cleanest is to weigh three of the Unknowns against three of the Known Neutrals.

Again, either the scale will stay balanced, or it will tip to the right, or it will tip to the left. If the scale stays balanced, then we know that the "moneta" we are looking for is the only one Unknown that remains off the scale. To figure out if it is light or heavy, simply weigh it against a Known Neutral and we have our answer.

But if the scale tips on this second weighing, then we have three possible "moneta" . If it tips so that the Unknowns go down, then one of those three "moneta" is the odd one out, and it's heavy. If it tips the other way, than one of the three suspect "moneta" is light.

Let's assume that the scale tips down on the side with the Unknowns, which means they become Heavy Suspects. We have three Heavy Suspects, and the rest Known Neutrals, so we can weigh two of the Heavy Suspect balls against each other, and if the scale tips, the "moneta" we are looking for is the heavier ball, and if it doesn't, the third Heavy Suspect that we did not include in the third weighing is the "moneta" , and it is heavy. (The same is true if the second weighing gives you three Light Suspects, simply inverted).

The solution, assuming the first weighing stays balanced, will look something like this:



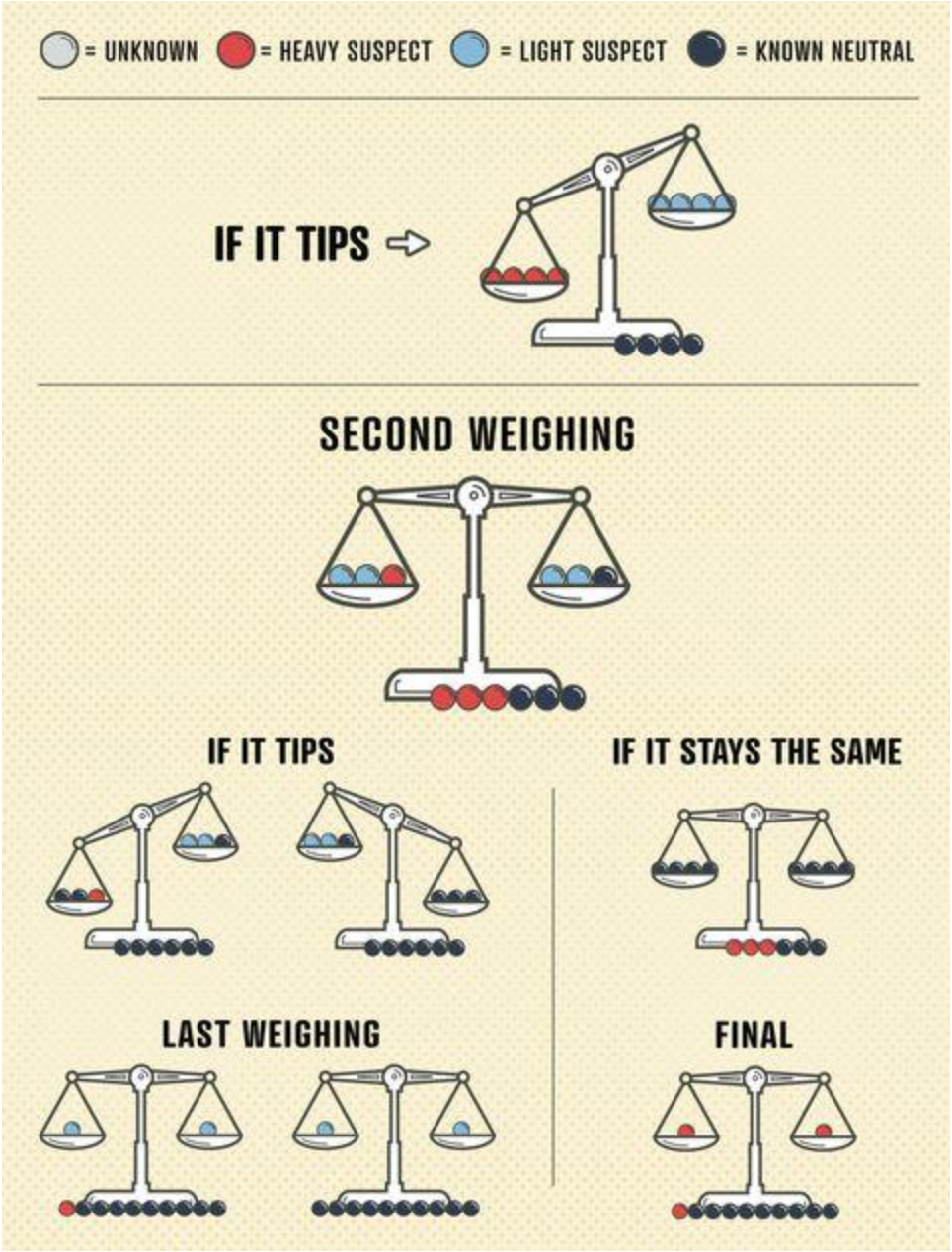
Now we have to solve for the other possibility—that the scale tips on the first weighing of four vs. four. In this case, you have four Heavy Suspects, four Light Suspects, and four Known Neutrals (the four that remained off the scale for the first weighing you know to be neutral). Again, there are multiple ways you can set up the second weighing to arrive at the solution. Here is one.

Weigh two Light Suspects and a Heavy Suspect against the other two Light Suspects and a Known Neutral. The scale will either tip or stay balanced. If it stays balanced, then you know that the odd ball you're looking for is one of the three Heavy Suspects that stayed off the scale. Like before, you can then weigh one of them against another. If it tips, it's the heavier ball. If it doesn't, it's the "moneta" remaining off the scale during the third weighing.

What if the scale tips on the second weighing (the Light-Light-Heavy vs. Light-Light-Neutral weighing)? If it tips to the right, so the Light-Light-Neutral side goes down, then you know it must be one of the Light Suspects on the other side of the scale—the left side in this example, the side that went up. You can weigh them against each other to figure out which one is the light ball.

But, if it tips the other way, so the Light-Light-Heavy side on the left goes down, then you still have three suspects: the two Light Suspects on the right side that went up, and the Heavy Suspect on the left side that went down. For the third and final weighing, weigh the two Light Suspects against each other. If the scale tips, the lighter ball is the odd ball you've been looking for. If it stays balanced, then you know the odd ball is the Heavy Suspect that stayed off the scale for the third weighing.

If you invert heavy and light suspects in the second weighing, the solution is the same. It will look something like this:



Now, as I said, there are more solutions to this problem. For example, if the scale stays balanced on the first weighing, then on the second weighing, you can weigh Unknown-Unknown-Neutral vs. Unknown-Neutral-Neutral, and still arrive at the solution. Similarly, if the scale tips on the first weighing, then you can measure a combination of four vs. four again to arrive at the solution: Light-Neutral-Neutral-Neutral vs. Heavy-Light-Light-Light, or the inversion.

There was no correct solution to problem 16

Rules

1. Anyone is eligible to participate. Each solution is to be the work of one individual without any input from faculty or others. An answer must be accompanied by appropriate justifications to be considered correct.
2. The solution is to be submitted with the solver's name, email, year in school (if applicable), local phone number, and local address. If you are submitting this for possible credit in a class, include your class number and instructors name.
3. The solution is to be typed or legibly written. Solutions must be submitted to the by 2 p.m. on the due date.
4. Entries will be graded on clarity of exposition and elegance of solution. An award of GEL10 will be given for the best correct solution. In
the case of a two-way tie, the award will be split. If there are more than two best solutions, a drawing will be held to determine two award winners.
5. Graduate students, faculty, and members of the general public are encouraged to submit solutions, but they will not be considered.

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Solution for this problem can be submitted proveweek@gmail.com