



Monday cup #12- Solution

Posted on: June 17, 2019

Due on: June 23, 2019



Problem

Lower Elementary:

Question: $1 - 2 + 3 - 4 + \dots - 98 + 99 =$

Answer: 50

Solution:

Solution 1

If we group consecutive terms together, we get $(-1) + (-1) + \dots + 99$, and since there are 49 pairs of terms the answer is $-49 + 99 = 50 \Rightarrow (E)$.

Solution 2

(Similar to Solution 1) If we rearranged the terms, we get $1 + 3 - 2 + 5 - 4 \dots + 99 - 98$ then $1 + 1 + \dots + 1$, and since there are 49 pairs of terms and the 1 in the beginning the answer is $1 + 49 = 50 \Rightarrow (E)$.

Solution 3

Let $1 - 2 + 3 - 4 + \dots - 98 + 99 = S$.

Therefore, $S = 99 - 98 + 97 - \dots - 4 + 3 - 2 + 1$

We add:

$$2S = 100 - 100 + 100 - 100 \dots + 100 = 100$$

$$S = 50 \Rightarrow (E)$$

Solution 4

We proceed with addition, $1 - 2 + 3 - 4 \dots$. Once done we find 50

Upper Elementary:

Question: : (italiano) PROBLEMA NUM: 136 - CATEGORIA: generic---

<https://www.pianetaproblemi.it/problema-136-generici.php>

Answer: 153

Solution:

PROBLEMA NUM: 136 - CATEGORIA: generici

Carlo ha pagato un debito di euro 38, un altro di euro 146 e un terzo di euro 275.

Quanti euro ha pagato in tutto?

TUO SVOLGIMENTO:

153

TUA RISPOSTA:

153

SVOLGIMENTO CORRETTO:

$38+146+275 =$ euro 459

RISPOSTA CORRETTA:

Carlo ha pagato in tutto euro 459

Middle School:

Question:

Trois hommes ont à partager 21 tonneaux sept pleins à demi Je demande comment se peut faire le partage, en sorte que tous trois aient un égal nombre de tonneaux, et égale quantité de vin

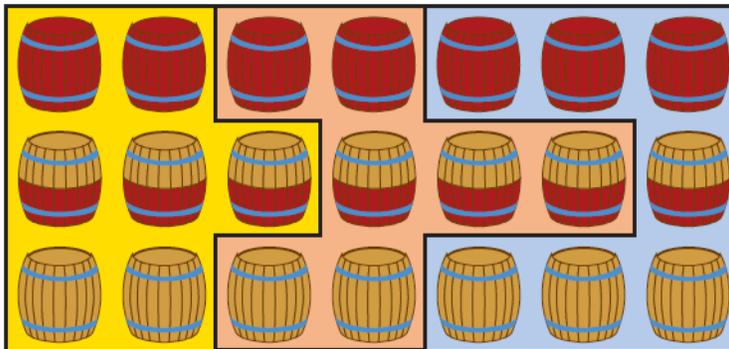


Answer:

Solution:

On peut évidemment trouver la solution sans aucune mathématique.

En voici une image :



Et chacun se retrouve bien avec 7 tonneaux et le contenu de 3,5 tonneaux en vin.

En fait, le problème est assez simple à résoudre, même dans le cas général de $3n$ tonneaux (n pleins, n à moitié pleins et n vides) : il suffit de décomposer n en une somme de 3 nombres chacun inférieur à $n/2$:

$$n = a + b + c \left(a \leq \frac{n}{2}, b \leq \frac{n}{2}, c \leq \frac{n}{2} \right).$$

Une solution est alors :

la première personne prend
 a pleins, a vides et $n - 2a$ demi pleins.

la deuxième personne prend
 b pleins, b vides et $n - 2b$ demi pleins.

la troisième personne prend
 c pleins, c vides et $n - 2c$ demi pleins.

On voit ainsi que la solution donnée ci-dessus correspond à la décomposition $7 = 2 + 2 + 3$, mais qu'il y a une deuxième solution correspondant à $7 = 1 + 3 + 3$.

Algebra and Up:

Question:

Define $[a, b, c]$ to mean $\frac{a+b}{c}$, where $c \neq 0$. What is the value of

$$[[60, 30, 90], [2, 1, 3], [10, 5, 15]]?$$

Answer: 2

Solution:

$$\text{Note that } [ta, tb, tc] = \frac{ta + tb}{tc} = \frac{t(a+b)}{tc} = \frac{a+b}{c} = [a, b, c].$$

$$\text{Thus } [60, 30, 90] = [2, 1, 3] = [10, 5, 15] = \frac{2+1}{3} = 1, \text{ and } [1, 1, 1] = \frac{1+1}{1} = 2.$$

**There were correct solutions from Gigi Zakaradze (Georgia, the country).
The prize was split between Zakaradze**

Rules

1. Anyone is eligible to participate. Each solution is to be the work of one individual without any input from faculty or others. An answer must be accompanied by appropriate justifications to be considered correct.
2. The solution is to be submitted with the solver's name, email, year in school (if applicable), local phone number, and local address. If you are submitting this for possible credit in a class, include your class number and instructors name.
3. The solution is to be typed or legibly written. Solutions must be submitted to the by 2 p.m. on the due date.
4. Entries will be graded on clarity of exposition and elegance of solution. An award of **GEL10** will be given for the best correct solution. In the case of a two-way tie, the award will be split. If there are more than two best solutions, a drawing will be held to determine two award winners.
5. Graduate students, faculty, and members of the general public are encouraged to submit solutions, but they will not be considered.

**კრძაბათის თასი, кубок понедельника, Monday cup, Coppa del lunedì, Coupe du lundi
Solution for this problem can be submitted proveweek@gmail.com**