Monday cup #10– Solution

Problem

Lower Elementary:

Question: Ten years ago   Keda (Georgia, the country),had a population of approximately 8000. Jessie knew the digits 3, 7, 8, 9 were part of the actual population at that time, but could not remember the order of the digits. What might the actual population of Keda (Georgia, the country), have been if each digit is used once?

Upper Elementary:

Question: Nino takes her two cats to the veterinarian in a pet carrier which weighs 2.1 kg. Her older cat, Brinker, weighs 1 kg more than her younger cat, Skates.

When the vet weighs both cats in their carrier, the scale registers 11.3 kg.

How much does each cat weigh?

Middle School:

Question: In the diagram, P is the point (6, −4)   and   O is the origin . The path length from O to P is 10. How many points with integer coordinates have a path length of 10 from O?

Algebra and Up:

Question: A tortoise and a hare begin a 4-mile race at the same time. The tortoise travels 2 miles per hour, and the hare travels 24 miles per hour. After the hare races for 8 minutes, he stops to take a nap. The tortoise passes the hare and wins the race. How much time does it take for the tortoise to reach the finish line after it passes the hare?

There were correct solutions from   Matthew Fitzpatrick   (U.S. state of Mississippi).

Rules

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Solution for this problem can be submitted proveweek@gmail.com

Lower Elementary:

Question: Ten years ago Keda (Georgia, the country),had a population of approximately 8000. Jessie knew the digits 3, 7, 8, 9 were part of the actual population at that time, but could not remember the order ofthe digits. What might the actual population of Keda (Georgia, the country), have been if each digit is used once?

Solution:

If we are looking for a number that is close to 8000, the best options would start with either an 8 or a 7. Using the digits 3; 7; 8; 9 each exactly once we will find the number closest to 8000 but greater, and then find the number closest to 8000 but smaller.

Closest Number to 8000 but greater

For numbers that start with an 8, the smallest value would have the hundreds digit as small as possible, the tens digit would be the next smallest number, and ones digit would be the largest number. This means that the number starting with 8 that is closest to 8000 but greater and uses each of the digits 3, 7, 8, 9 is 8379. This number is almost 400 more than 8000.

Closest Number to 8000 but smaller

For numbers that start with a 7, the largest value would have the hundreds digit as large as possible, the tens digit would be the next largest number and the ones digit would be the smallest number. This means that the number starting with 7 that is closest to 8000 but smaller and uses the digits 3, 7, 8, 9 is 7983. This number is 17 less than 8000.

If we were looking for the number closest to 8000 that uses the digits 3, 7, 8, 9, we would select 7983.

However, we were not asked for the closest number to 8000 but rather a number that could be reasonably approximated by 8000. In one case, we could argue either of 7938 or 7983 since they are both within 100 of 8000.

We could also argue the numbers 7839 or 7893 or 7938 or 7983 or 8379 or 8397 since each is within 400 of 8000 and could be reasonably rounded to 8000.

Each of the above arguments could act as justification for your answer.

Teacher's Notes

The numbers 7983, 7938, and 8379, or even 8397, 7893 and 7839 are all reasonable choices as a population of   Keda (Georgia, the country),   since the result of rounding these numbers to the nearest 1000 gives us 8000 in each case. As quantitative values get bigger, we tend to care less about the precise digits of those values, and are interested mostly in the most significant digits.

Calculators are only able to display a limited number of digits. If the calculated values get too big or too small they are usually shown in scientific notation or E-notation. Numbers written in this format show some number of digits multiplied by a power of 10. For example, the number of atoms in a mole is approximately $6.022 \_ 10^{23}$, and the mass of an electron is approximately $9.10938 \_ 10\square31$ kg. The precision of a number written in scientific notation is determined by the number of significant digits. The mole size has four significant digits and the mass has six significant digits.

In some situations we might not care about precision at all. We may want to have a general sense of the size of some measured value in terms of its order of magnitude. In these cases, we are interested in comparing values to the closest power of 10. For example, consider the population of Austria (approximately 8.7 million in 2016) and the population of India (approximately 1.3 billion in 2016). We are unlikely interested in comparing the differences in these numbers by subtracting them. We can say that the order of magnitude difference between the population of India and Austria is 2. This means that the population of India is approximately 10^2 or 100 times that of the population of Austria. However, if we compare the population of Canada (approximately 36 million in 2016) to the population of the United States (approximately 323 million in 2016) we could say that the order of magnitude difference between the population of the United States and Canada is 1. These descriptions give us a broad sense of the relative sizes of the populations.

Question: Nino takes her two cats to the veterinarian in a pet carrier which weighs 2.1 kg. Her older cat, Brinker, weighs 1 kg more than her younger cat, Skates.
When the vet weighs both cats in their carrier, the scale registers 11.3 kg.
How much does each cat weigh?

Solution:
We know that the combined weight of the two cats is equal to the total weight of the two cats and the carrier minus the weight of the carrier. That is, the combined weight of the two cats is 11,3-2,1 = 9,2 kg.
The 9:2 kg is made up of Shorty's weight plus Chuckles' weight. If Chuckles were 1 kg lighter, he would be the same weight as Shorty. So two times the weight of Shorty would be 9,2 - 1 = 8,2 kg.
Then, Shorty's weight would be 8,2 : 2 = 4,1 kg.
Since Chuckles weighs 1 kg more than Shorty, Chuckles weighs 4,1 + 1 = 5,1 kg.
We can verify our answer by adding Shorty's weight, Chuckles' weight and the weight of the carrier. Then, 4,1 + 5,1 + 2,1 = 11,3 kg, the given total weight.
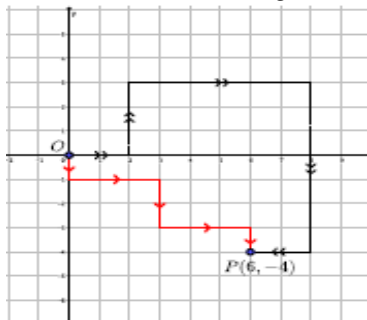Therefore, Shorty weighs 4,1 kg and Chuckles weighs 5,1 kg.

Middle School:

Question: In the diagram, P is the point $(6, -4)$ and O is the origin. The path length from O to P is 10. How many points with integer coordinates have a path length of 10 from O?

Solution:
Let Q(a, b) be a point that has path length of 10 from O, the origin.
Let's first assume that Q is on the x or y axis.

P(6, -4)

The only point along the positive x-axis that has path length 10 from the origin is (10, 0).
The only point along the negative x-axis that has path length 10 from the origin is (-10, 0).
The only point along the positive y-axis that has path length 10 from the origin is (0, 10).

The only point along the negative y-axis that has path length 10 from the origin is (0,-10).
Therefore, there are 4 points along the axes that have a path length of 10 from O.
Next, let's assume a > 0 and b > 0, so Q is in the first quadrant.
Since the path length from O to Q is 10, there must be a path from O to Q that moves a total of r units to the right and u units up (in some order) such that r + u = 10. This means that Q is r units to the right of O and u units up from O. In other words, a = r and b = u, so
a + b = r + u = 10.
The points (a, b) in the first quadrant that satisfy a + b = 10 where a and b are integers are (1, 9); (2, 8); (3, 7); (4, 6); (5, 5); (6, 4); (7, 3); (8, 2); (9, 1). There are 9 such pairs. Therefore, there are 9 points in the first quadrant that have path length of 10 from O.
By symmetry, there are 9 points in each of the four quadrants that have path length of 10 from O.
In quadrant 2, the points are
(-1, 9); (-2; 8), (-3, 7); (-4; 6); (-5,5); (-6, 4); (-7, 3); (-8, 2); (-9, 1):
In quadrant 3, the points are
(-1,-9); (-2,-8); (-3,-7); (-4,-6); (-5,-5); (-6,-4); (-7,-3); (-8,-2); (-9,-1):
In quadrant 4, the points are
(1,-9); (2,-8); (3,-7); (4,-6); (5,-5); (6,-4); (7,-3); (8,-2); (9,-1):
Therefore, there are a total of 4+(4X9) = 40 points with integer coordinates that have a path length of 10 from O.

Algebra and Up:

Question: A tortoise and a hare begin a 4–mile race at the same time. The tortoise travels 2 miles per hour, and the hare travels 24 miles per hour. After the hare races for 8 minutes, he stops to take a nap. The tortoise passes the hare and wins the race. How much time does it take for the tortoise to reach the finish line after it passes the hare?

Answer: 24 minutes

Solution: First, we need to know how far the hare gets in 8 minutes (or 2/15 of an hour): 24 mph × 2/15 hr = 16/5 miles. That means that there are 4/5 of a mile left to go for the tortoise after it passes the hare. To find the amount of time, we solve for t: 4/5 of a mile = 2 mph × t, so t = 2/5 of an hour, or 24 minutes.